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THE CALCULATION OF HEAT TRANSFER TO THICK METAL WALLS OF STEAM TURBINE COMPONENTS FROM AMBIENT STEAM WITH TIME-DEPENDENT CHARACTERISTICS

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Abstract—The paper treats non-steady heat conduction, normally through a parallel-sided slab and radially through a cylinder when the rear-side of the slab, or outside of the cylinder, is perfectly insulated, and when the front-side of the slab, or inside of the cylinder, is subjected to a flowing vapour whose pressure, temperature, and mass flow rate vary with time in some prescribed manner. The problem is typical of starting conditions for steam turbines. Formulae are assumed for the vapour-side coefficients (both with and without condensation), and finite-difference approximations are used to obtain solutions to the conduction problem.

Certain questions are raised concerning the applicability of the extant vapour-side coefficients, which have been derived empirically under steady-state, one-dimensional conditions, for the solution of non-steady heat conduction problems of the type under consideration.

NOMENCLATURE

C_p, specific heat of solid [Btu/(lb degF)];

C_s, specific heat of steam at constant pressure [Btu/(lb degF)];

C_t, $= \dfrac{\partial T_G/\partial t}{T_G - T_0}$;

D, diameter of heated wall surface [ft];

G, $= w/s$;

h_t, $q/(T_G - T_S)$, heat-transfer coefficient for forced convection [Btu/(h ft^2 degF)];

h_c, $q/(T_{sv} - T_S)$, heat-transfer coefficient for film-type condensation [Btu/(h ft^2 deg F)];

H_t, $= \begin{cases} \dfrac{2\Delta x \cdot L}{k} \cdot h_t, & \text{in Cartesian co-ordinates;} \\ \dfrac{2\Delta R \cdot r_i}{k} \cdot h_t, & \text{in cylindrical co-ordinates;} \end{cases}$

k, thermal conductivity of solid [Btu/(h ft degF)];

L, total distance over which heat flow in the solid takes place [ft];

p, steam pressure [lbf/in^2, absolute];

q, heat flow [Btu/(h ft^2)];

r, radial distance measured in the direction of heat flow [ft];

r_i, radius of inner, heated surface [ft];

r_e, radius of outer, insulated surface [ft];

R, $= r/r_i$;

R_i, $= 1$;

R_e, $= r_e/r_i$;

S, $= \pi D^2/4$;

T, temperature [degF];

T_G, ambient steam temperature [degF];

T_0, initial temperature of solid at zero time [degF];

T_S, heated surface temperature [degF];

T_{Sv}, saturation temperature [degF];

t', time [h];

t, $= \begin{cases} t'/(\rho c_p/k)L^2, & \text{non-dimensional time in Cartesian co-ordinates;} \\ t'/(\rho c_p/k)r_i^2, & \text{non-dimensional time in cylindrical co-ordinates;} \end{cases}$

w, mass flow of steam [lb/h];

x', distance measured in the direction of heat flow [ft];
x, $= x'/L$.

Greek symbols

β, $= \begin{cases} \Delta t/(\Delta x)^2, \text{ in Cartesian co-ordinates;} \\ \Delta t/(\Delta R)^2, \text{ in cylindrical co-ordinates;} \end{cases}$

ΔR, non-dimensional finite-difference net width;
Δt, finite-difference net width in non-dimensional time;
Δx, non-dimensional finite-difference net width;
ρ, density of the solid [lb/ft^3];
$\phi_{j,t}$, $= \dfrac{T - T_0}{T_G - T_0}$, non-dimensional temperature, in which the first subscript j denotes the geometric location and the second subscript t denotes the time location on the finite-difference network.

INTRODUCTION

DURING the periods of heating and cooling of steam turbine components, which occur when the procedures for start-up and shut-down are in operation, time-dependent temperature variations take place with accompanying temperature gradients throughout the body metal. These gradients can induce stresses and strains which are significant for the strength, deformation, and life of the turbine.

This problem is essentially one of non-steady transfer of heat from the ambient steam to the metal component under consideration when the steam has the time-dependent characteristics of temperature, pressure, and mass flow.

The mechanism of heat transfer from vapour to metal is usually envisaged as consisting of: (i) the transfer of heat from the vapour to the metal surface which is in contact therewith; (ii) the conduction of heat through the metal. The available information concerning the vapour-side transfer of heat is largely experimental in origin, and is expressed by Newton's formula:

$$q = h(T_G - T_S).$$

This formula is used as a boundary condition in mathematical formulations.

In cases of condensation and forced convection, the unit surface conductance, or heat-transfer coefficient, h takes on complicated forms which embody non-linear characteristics, such as vapour-side surface temperature; component geometry; mass flow, pressure, and temperature of the vapour; and when condensation prevails, on characteristics of the condensate film or droplets. A wide variety of such forms are presented in, for example, reference [1].

In the non-steady heat-transfer problem, in which the vapour characteristics have given time-dependent variations, it follows that the coefficient h (now denoted by h_t) will also be time-dependent. Thus, the analysis of the conduction of heat through the metal will be governed by the coefficient h_t and by the metal component characteristics, and will require the determination of consistent values of h_t at all times during the period of non-steady transfer of heat from the vapour. Once values of h_t are known, the temperature gradients and their time-history can readily be evaluated throughout the metal.

In the work to follow the term "heat-transfer coefficient" is intended to signify the coefficient h_t as discussed in the foregoing remarks.

Due to the arbitrary nature of the given vapour characteristics in so far as mathematical description thereof is concerned, the analysis of the non-steady heat conduction problem must take the form of numerical approximations.

PURPOSE OF ANALYSIS

The purpose of the analysis to follow is to provide a numerical method of determining the non-steady temperature gradients through the metal of typical steam turbine components during periods of start-up. To accomplish this, it is necessary to account for the time variation of the heat-transfer coefficient h_t, as described in the Introduction. It is also required to determine when the heat-transfer process changes from one type of mechanism to another, e.g. from condensation to forced convection.

Lacking other information, the empirical formulae for the transfer of heat from the steam to the metal, as available in the literature (cf. [1]), will be used for the purpose, although these formulae have been devised basically for heat transfer in one dimension and under steady-state conditions. These aspects of the problem will be discussed later.

METHOD OF ANALYSIS

Because the available information on heat transfer is mainly confined to heat flow in one spatial dimension, the geometric types of component which can be used to illustrate the method of analysis must be limited. For present purposes, the analysis to follow will be confined to the geometric forms of cylinder and slab, for which a differential equation formulation is appropriate, and in which the boundary conditions for the metal components comprise heat input from the vapour-side, in accordance with Newton's formula, and complete insulation at the outside surface. The Fourier Heat Conduction equation, reduced appropriately for heat flow in one dimension, together with the appropriate boundary conditions, can be expressed in the following non-dimensional forms:

In Cartesian co-ordinates,

$$\left.\begin{aligned} \frac{\partial^2\phi}{\partial x^2} &= \frac{\partial\phi}{\partial t} + C_t\phi \\ \frac{\partial\phi(0,t)}{\partial x} &= \frac{Lh_t}{k}\left[\phi(0,t) - 1\right] \\ \frac{\partial\phi(1,t)}{\partial x} &= 0 \\ \phi(x,0) &= 0\,. \end{aligned}\right\} \quad (1)$$

In cylindrical co-ordinates,

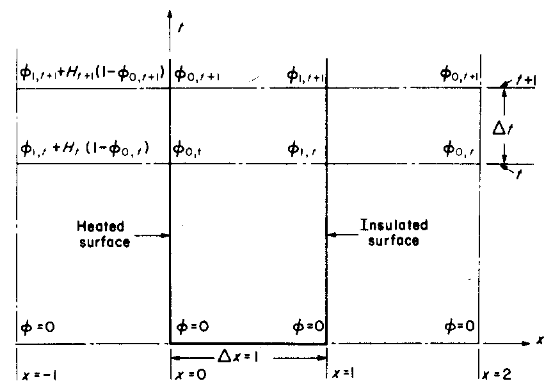
$$\left.\begin{aligned} \frac{\partial^2\phi}{\partial R^2} + \frac{1}{R}\frac{\partial\phi}{\partial R} &= \frac{\partial\phi}{\partial t} + C_t\phi \\ \frac{\partial\phi(1,t)}{\partial R} &= \frac{r_i h_t}{k}\left[\phi(1,t) - 1\right] \\ \frac{\partial\phi(R_e,t)}{\partial R} &= 0 \\ \phi(R,0) &= 0\,. \end{aligned}\right\} \quad (2)$$

These equations are approximated to by a finite-difference network of rectangular meshes the length of one side of which equals the non-dimensional time interval Δt and that of the other side equals the non-dimensional spatial interval Δx or ΔR. By increasing the spatial interval so that it is equal to the whole distance between the heated and insulated boundary surfaces, expressions can readily be derived which give explicitly non-dimensional temperatures ϕ at these two surfaces. In spite of the coarseness of this network, the surface temperatures calculated therefrom would seem to be adequate for evaluating the required heat-transfer coefficients.

Having obtained the transfer coefficients in this manner, they can be introduced into the boundary conditions of a standard numerical computation, with a suitable number of mesh points arranged internally throughout the metal thickness, and based on the Fourier equation. From this computation the heated and insulated surface temperatures in particular will have been obtained, so that a check on the values of the heat-transfer coefficients originally estimated can be made by comparing the two sets of surface temperatures. This comparison will indicate whether or not a further iteration is required.

The governing equations (1) and (2), being of the parabolic type, can give rise to unstable numerical computations when approximated by finite-differences. Suitable finite-difference forms which mitigate this instability are given in the literature (e.g. [2, 3]). Their application to systems (1) and (2) using an "explicit–implicit" form gives the networks and approximations summarized in Figs. 1 and 2. After insertion of the boundary and initial conditions indicated on Figs. 1 and 2, the finite-difference expressions for the internal equations are applied to a typical pivotal point on each of the heated and insulated boundaries. Two simultaneous equations arise, containing as unknowns two non-dimensional temperatures at typical times. The equations are resolved to give recurrence formulae for $\phi_{j,t+1}$ which can be evaluated explicitly therefrom. These recurrence formulae are summarized in Table 1.

The procedure is now to apply the recurrence formulae, starting at the first time interval after zero. The particular form of heat transfer (e.g. forced convection, condensation) at the input surface at this time must first be assumed. The validity of this assumption can be checked after evaluation of the surface temperatures.



Internal Equation

$$\beta\phi_{j-1,t+1} - 2(\beta + 1)\,\phi_{j,t+1} + \beta\phi_{j+1,t+1} = -\beta\phi_{j-1,t} + 2(\beta - 1 + C_t\cdot\Delta t)\,\phi_{j,t} - \beta\phi_{j+1,t}$$

Heated Surface Boundary

$$\phi_{-1,t} = \phi_{1,t} + H_t(1 - \phi_{0,t})$$

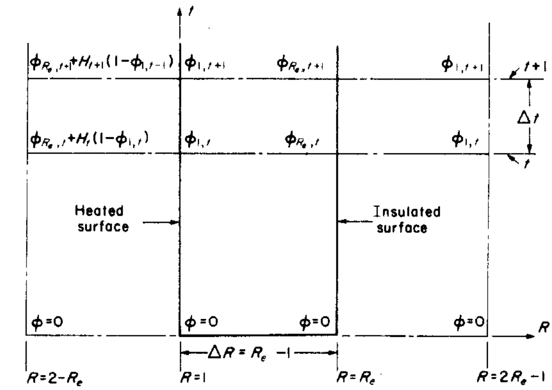
Insulated Surface Boundary

$$\phi_{2,t} = \phi_{0,t}$$

Initial Condition

$$\phi_{j,0} = 0$$

FIG. 1. Finite-difference network and approximations in Cartesian co-ordinates.



Internal Equation

$$\beta\phi_{j-1,t+1} - 2(\beta + 1)\,\phi_{j,t+1} + \beta\phi_{j+1,t+1} = \left(\frac{\Delta t}{\Delta R}\cdot\frac{1}{R_j} - \beta\right)\phi_{j-1,t} + 2(\beta - 1 + C_t\cdot\Delta t)\,\phi_{j,t} - \left(\frac{\Delta t}{\Delta R}\cdot\frac{1}{R_j} + \beta\right)\phi_{j+1,t}$$

Heated Surface Boundary

$$\phi_{(2-R_e),t} = \phi_{R_e,t} + H_t(1 - \phi_{1,t})$$

Insulated Surface Boundary

$$\phi_{(2R_e-1),t} = \phi_{1,t}$$

Initial Condition

$$\phi_{R,0} = 0$$

FIG. 2. Finite-difference network and approximations in cylindrical co-ordinates.

In the case of film-type condensation, for example, the heat-transfer coefficient is a non-linear function of surface temperature. However, the surface temperature cannot be determined without knowing the values of the transfer coefficients. In order to overcome this difficulty, a suitable range of surface temperature T_S is assumed. For several values of T_S within the range, ϕ-values at the heated surface are calculated using the expression

$$\phi = \frac{T_S - T_0}{T_G - T_0}.$$

Also, values of h_t are calculated from the selected transfer formula for film condensation, using the assumed values of T_S. Note that h_c may be defined in terms of saturation temperature T_{S_v} so that h_c must be converted, for consistency, into terms of steam ambient temperature T_G.

Table 1. Recurrence formulae

Coordinate system	Recurrence formula
Cartesian	$\phi_{0,t+1} = \dfrac{\beta(\beta + 1)(H_t + H_{t+1}) + [2 - \beta(\beta + 1)\,H_t - 2(\beta + 1)\,C_t \cdot \Delta t]\,\phi_{0,t} + 2\beta(2 - C_t \cdot \Delta t)\,\phi_{1,t}}{2 + 4\beta + \beta(\beta + 1)\,H_{t+1}}$
	$\phi_{1,t+1} = \left(\dfrac{\beta}{\beta + 1}\right)\phi_{0,t+1} + \left(\dfrac{\beta}{\beta + 1}\right)\phi_{0,t} + \left(\dfrac{1 - \beta - C_t \cdot \Delta t}{\beta + 1}\right)\phi_{1,t}$
Cylindrical	$\phi_{1,t+1} = \dfrac{(\beta + 1)[(\beta - \Delta t/\Delta R)\,H_t + \beta H_{t+1}] + [2 - (\beta + 1)(\beta - \Delta t/\Delta R)\,H_t - 2(\beta + 1)\,C_t \cdot \Delta t]\,\phi_{1,t} + 2\beta(2 - C_t \cdot \Delta t)\,\phi_{R_e,t}}{2 + 4\beta + \beta(\beta + 1)\,H_{t+1}}$
	$\phi_{R_e,t+1} = \left(\dfrac{\beta}{\beta + 1}\right)\phi_{1,t+1} + \left(\dfrac{\beta}{\beta + 1}\right)\phi_{1,t} + \left(\dfrac{1 - \beta - C_t \cdot \Delta t}{\beta + 1}\right)\phi_{R_e,t}$

Thus, by equating heat-flow rates,

$$h_t = h_c \frac{T_{Sv} - T_S}{T_G - T_S}.$$

The h_t-values so obtained are re-expressed as H_t-values and inserted into the recurrence formulae from which is calculated a second set of ϕ-values. Both sets of ϕ-values can be plotted against T_S, and the intersection of the two curves gives the required values of T_S, and hence of ϕ, at both boundary surfaces, together with the corresponding value of H_t. The value of T_S so obtained is compared with the saturation temperature T_{Sv} of the steam at the corresponding time t to confirm whether or not the assumption of a condensation condition was correct, i.e. $T_{Sv} > T_S$ gives condensation. If this assumption is incorrect, then h_t must be re-evaluated at this time from the selected formula for the alternative heat-transfer mechanism envisaged as appropriate for the case under analysis.

At subsequent time intervals, this calculation procedure is repeated, using the appropriate parameters evaluated at the previous time interval, and so on.

In general, although attention has been focussed particularly on film condensation and forced convection processes, nevertheless given sufficient experimental and other information, other types of heat transfer, such as drop-wise condensation, boiling, natural convection, radiation, can be incorporated in the method with appropriate adjustments to the procedure.

NUMERICAL EXAMPLE

To illustrate the calculation procedure, the case of a circular tube of uniform wall thickness is analysed under cold start transient conditions. These conditions comprise at zero time stationary steam at 450°F and a uniform temperature of 70°F throughout the metal. The steam is assumed to have a single component of flow inside the tube in the axial direction, and the steam conditions are assumed to be uniform over the tube inside-wall surface and to have characteristics with the time histories given in Fig. 3. The outside surface of the tube is taken to be completely insulated. The forms of heat transfer from steam to tube are taken as film-type condensation and forced convection.

All quantities are expressed in the units: lb, ft, h, degF, Btu.

Cylinder characteristics:

$r_i = 1.979$	$R_i = 1$
$r_e = 2.292$	$R_e = 1.1579$
$D = 3.958$	$\Delta R = 0.1579$
$S = 12.3$	$\rho = 488.9$
$T_0 = 70$	$C_p = 0.11$
$k = 18.2$	
$t = t'/11.6$	$H_t = 0.0344 h_t$

C_t values:

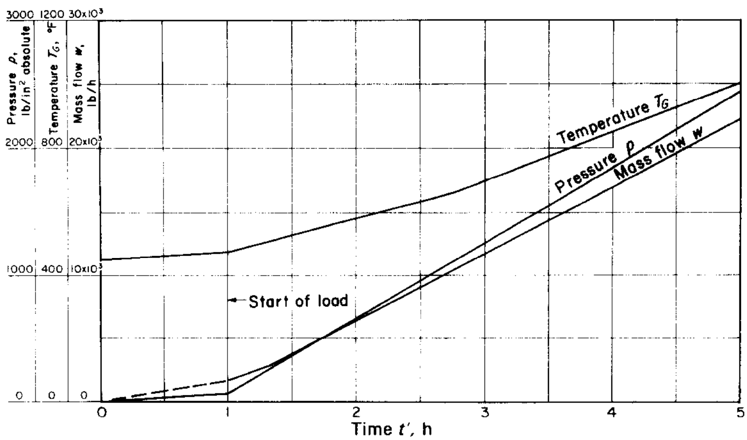


FIG. 3. Time-histories of steam conditions for numerical example.

To simplify calculations the following mean values of C_t are used:

Range of t	C_t
0–0·0862	0·740
0·0862–0·2430	2·500
0·2430–0·4310	2·350

Heat-transfer coefficient formulae:

For film-type condensation, formula (13–12)* of reference [1] is used and re-expressed in the following form:

$$h_c = 0{\cdot}725(\psi_f)^{\frac{1}{4}} \,.\, (\alpha_f)^{\frac{1}{4}},$$

where: $\psi_f = \dfrac{k_f^3 \rho_f^2 g}{\mu_f^2}$;

$$\alpha_f = \frac{\lambda \mu_f}{D \cdot \Delta T};$$

k_f is thermal conductivity of condensate [Btu/(h ft degF)];
ρ_f is density of condensate film [lb/ft^3];
μ_f is absolute viscosity of condensate film [lb/(h ft)];
λ is enthalpy change [Btu/lb];
$\Delta T = T_{Sv} - T_S$;
$g = 4{\cdot}17 \times 10^8$ [ft/h^2];

and quantities are evaluated at temperature $T_f = 0{\cdot}25\, T_{Sv} + 0{\cdot}75 T_S$.

For forced convection, formulae (9–15)* and (9–13)* [1] are used as follows:

$$h_\infty = 0{\cdot}0144 C_S G^{0\cdot8}/D^{0\cdot2},$$

$$h_t/h_\infty = 1 + (D/L_T)^{0\cdot7},$$

where: h_∞ is at large values of L_T/D;
L_T is heated length of tube.

In the present case, the Reynolds numbers for the tube after the time anticipated for dry-wall conditions to prevail increase from approximately 50000, so that turbulent flow is well-developed. To account for this condition the following expression for h_t is used in the calculations:

$$h_t = 0{\cdot}02525 C_S G^{0\cdot8}/D^{0\cdot2}.$$

Time intervals Δt used in the calculation

Range of t	Δt
0–0·020	0·005
0·020–0·100	0·010
0·100 onwards	0·040

The evaluation from the above data of heat-transfer coefficients h_t, together with inside and outside surface temperatures, was performed by desk calculation. A computer calculation was also arranged to incorporate these h_t-values, and to have a mesh network with ten intervals through the wall thickness of the tube and to print out temperatures at all mesh points at time intervals of $t = 0{\cdot}005$ ($t' = 0{\cdot}058$ h). As a check on these results, the surface temperatures obtained in the process of calculating h_t-values are compared with the surface temperatures from the computer results, and plotted on Fig. 4 and 5. These graphs show a good correlation of temperatures and indicate that the values of heat-transfer coefficients as calculated by the approximate procedure presented here give reasonable results even without a second iteration.

An examination of the curve of Fig. 4 indicates that the time rate of heated surface temperature tends to drop as time t' approaches 1 h, but that after load increases again. Values of h_c, and consequently surface temperature, depend upon the steam saturation temperature T_{Sv}, which in turn is governed by the pressure p. The term $\psi_f^{\frac{1}{4}}$ in the expression for h_c, for example, has a reducing rate of increase with time before the "start of load" because the time rate of increase in T_{Sv} drops for the given constant time rate of pressure p. After the start of load, the time rate of the given pressure is increased so that the time rate of $\psi_f^{\frac{1}{4}}$ increases also. But

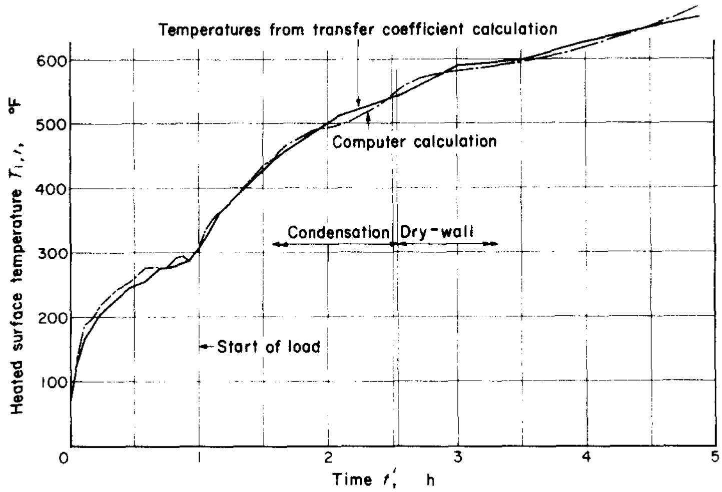


FIG. 4. Heated surface temperatures for numerical example.

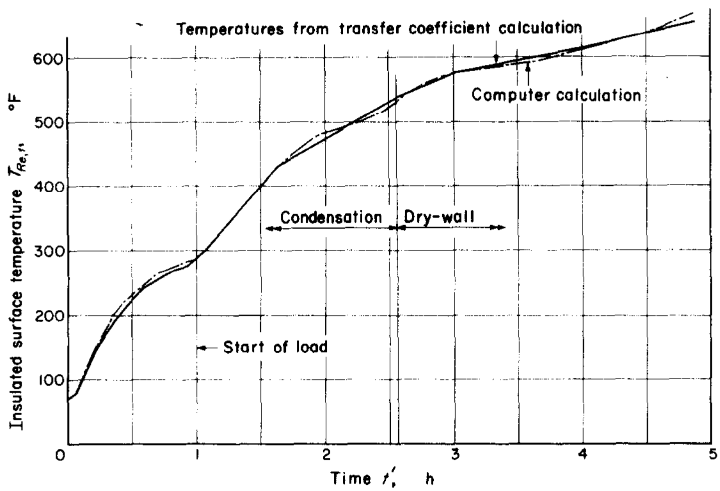


FIG. 5. Insulated surface temperatures for numerical example.

later, $\psi^{\ddagger}$ approaches its maximum value at $T_{Sv} \simeq 450$°F and drops thereafter. This reduction in $\psi_f^{\ddagger}$ occurs in the present example after $t' \simeq 1{\cdot}7$ h and leads eventually to dry-wall conditions.

It is likely that heat transfer by radiation from the steam will be significant at the higher temperatures of this example. However, the accompanying steam pressures are also high, and no information was available on radiation at high steam pressures. Consequently, these effects were not explored.

DISCUSSION

Several questions are implied by the foregoing work. The analysis as presented applies, lacking other information, the types of one-dimensional steady-state heat-transfer formulae as available in the literature, to the transfer of heat from the steam through thick metal components under transient conditions.

Although the present method does account for heat conduction through the metal in determining magnitudes of the transfer coefficients, particularly under conditions like condensation, the question may still be valid as to whether the heat rates through the metal would modify the form and variation of the empirical transfer formula itself. For example, would the same transfer formula be applicable to several tubes all with the same inside diameter and subjected to the same steam conditions, steady-state or transient, but with differing outside diameters?

The type of heat-transfer problem considered in this paper involves wide ranges of pressure, temperature, and mass flow. It may well be that, as a consequence, a corresponding extension of the range of applicability of the available heat-transfer formulae of the types already discussed is required to meet these conditions.

In practical design cases the geometries of turbine components are usually unsymmetrical or complicated in shape, and steam conditions vary spatially on heat input surfaces. The question then arises as to the validity of using the existing one-dimensional heat-transfer formulae for multi-dimensional configurations.

In general, from a practical standpoint these questions would be resolved if it could be shown that no significant errors would arise from the calculation of thermal stresses and strains based on the usual empirical, one-dimensional, steady-state heat-transfer formulae. As far as is known, no information has been published to elucidate these problems.

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Résumé—L'article traite de la conduction de la chaleur instationnaire, perpendiculairement à une plaque à faces parallèles et radialement à travers un cylindre lorsque la face arrière de la plaque ou l'extérieur du cylindre est parfaitement isolé et lorsque la face avant de la plaque ou l'intérieur du cylindre est soumis à un écoulement de vapeur dont la pression, la température et le débit massique varie au cours du temps d'une façon imposée. Le problème correspond aux conditions de démarrage pour les turbines à vapeur. On a choisi des formules pour les coefficients du côté de la vapeur (à la fois avec et sans condensation), et l'on emploie les approximations de différences finies pour obtenir les solutions du problème de conduction.

Certaines questions se posent à propos de l'applicabilité des coefficients encore existants du côté de la vapeur, qui ont été obtenus empiriquement avec des conditions stationnaires et unidimensionnelles, pour résoudre les problèmes de conduction de la chaleur non-stationnaires du type envisagé.

Zusammenfassung—Die Arbeit behandelt die instationäre Wärmeleitung quer in einem parallelseitigen Stab und radial in einem Zylinder, wobei das Ende des Stabes oder die Aussenseite des Zylinders perfekt isoliert sind und die Stirnseite des Stabes und die Innenseite des Zylinders einer Dampfströmung ausgesetzt sind, deren Druck, Temperatur und Drucksatz in vorgeschriebener Weise mit der Zeit veränderlich sind. Das Probelm ist charakteristisch für die Anlaufbedingungen von Dampfturbinen. Für die Koeffizienten auf der Dampfseite werden Formeln angenommen (sowohl mit als auch ohne Kondensation) und Näherungen mit endlichen Differenzen werden benützt um Lösungen des Leitungsproblems zu erhalten.

Einige Fragen wurden aufgeworfen über die Anwendbarkeit der dampfseitigen Koeffizienten, die empirisch unter stationären, eindimensionalen Bedingungen erhalten wurden und hier zur Lösung instationärer Wärmeleitprobleme dienen sollen.

Аннотация—В статье рассматривается нестационарный процесс передачи тепла теплопроводностью в направлении, перпендикулярном пластине с параллельными сторонами, и радиально через цилиндр, когда задняя стенка пластины или наружная поверхность цилиндра хорошо изолированы, а передняя стенка пластины или внутренняя поверхность цилиндра подвергаются действию потока пара, давление, температура и массовый расход которого изменяются со временем по заранее заданному закону. Задача типична для режима запуска паровых турбин. Предлагаются формулы для коэффициентов теплообмена между паром и поверхностью стенки (как для случая конденсации, так и без неё). Для получения решений задачи теплопроводности используются аппроксимации конечных разностей.

Рассматриваются вопросы применимости указанных коэффициентов, выведенных эмпирически при установившихся одномерных условиях, для решения рассматриваемого типа задач нестационарной теплопроводности.